Chapter 4:

Microeconomic foundations of the wage equation

4.1 Introduction

In this chapter I explore some individual preferences and constraints that may underlie the empirical relationships documented in Chapter 1.

Explaining why people may act in the way they do is necessary for welfare analysis. And it should provide guidance and motivation for further empirical work. For example, it may indicate conditions under which regressions would be unstable and it may indicate how variables should be specified. As with the interpretations discussed in Chapter 3, theoretical models can suggest other relationships, permitting corroboration, refutation and extensions.

There are three steps in this chapter. The first step is trivial. I respecify the wage equation of Chapter 1 in error-correction form, by including the lagged real wage as a regressor. This simplifies the particular interpretation I develop here but need not be appropriate for other possible interpretations. Specifically, it means that the wage data can be interpreted as gradual adjustment to a "target" or "steady state" level of real wages.

Second, I characterize this steady state. I develop a modified version of Pissarides' (1990) model of matching in the labor market that suggests that steady state real wages will be a positive function of the minimum wage, unemployment benefits, output per worker and a negative function of unemployment. The minimum wage and unemployment benefits affect wages because they represent outside options. Productivity matters because workers gain a share of the surplus from a match. Unemployment matters because it reflects the difficulty with which workers can be replaced.

Third, I explain why wages approach the steady state slowly. I develop a model of overlapping wage contracts in which predetermined wages form part of a workers outside opportunities. If wage bargains place a high weight on previously negotiated wages, then aggregate wages will respond sluggishly to shocks. The rate of adjustment depends on the distance between predetermined wages and the target. As unemployment is one element of the target, the rate of change of wages is a function of the unemployment rate. In this sense, the model explains the “Phillips curve”.

Explaining slow wage adjustment is the most important and difficult of these steps. It has implications that extend well beyond my econometric results. The bulk of the chapter is addressed to it.

Combining staggered contracts with a matching model predicts, in qualitative terms, many features of wage determination described in my regressions. It explains the determinants of the steady state and the nature of convergence towards this steady state. However, in other respects the model does not match the data well. In particular, the model suggests that wage adjustment may be noticeably quicker than implied by the regressions.

4.2 Description of the wage data

Modeling the wage equation is easier with some minor modifications to the specification used in Chapter 1. These involve little change to the fit of the equation or its statistical properties. They simplify the particular interpretation I develop here but are not necessarily appropriate for other possible interpretations.

Specifically, I put the equation in error-correction form, by including the lagged levels of real wages and productivity as discussed in Section 2.2.4. With the lagged levels of prices and wages both included in the right hand side, a rearrangement of variables can then express the minimum wage and unemployment benefits in real terms, rather than dividing by nominal wages.

The behavior of aggregate wages can then be described as follows:

$$\Delta w = \pi + .03\,[\,W^* - W]_{t-1} + \text{other terms} + \text{residual} \qquad (1)$$

where $$W^* = 0.73m/p + 0.24b/p + .04prod - 0.11U + \text{constant} \qquad (2)$$

w is the nominal wage

π is a moving average of lagged inflation

W^* is the (unobserved) target level of real wages, given by (2) (lagged one quarter)

W is the level of real wages (also lagged)

p is the level of consumer prices

m is the nominal minimum wage

b is the average level of unemployment benefit

prod is output per worker

U is the unemployment rate

"other terms" comprise seasonal factors, incomes policy and changes in payroll taxes, unemployment, productivity and minimum wages.

"the residual" is a small error that is approximately distributed normally and independently, with mean zero and standard error 0.0024

Data are quarterly. All variables except the unemployment rate are in logarithms. Further details regarding data measurement and specification are in the appendix to Chapter 1.

The equation implies the existence of a target or "steady state" level of real wages (denoted *W**). Wages adjust so as to remove 3 per cent of the deviation of real wages from their steady state level each quarter.

This specification is empirically indistinguishable in the short to medium run from equations that omit the level of real wages. Long term properties differ, but in ways that are subtle and, for many purposes, unimportant. In terms of Figure 2.3, it is an issue of whether the wage setting locus is vertical or very steep. In economic terms, it is an issue of whether wages take a long time or forever to adjust.

The objectives of this chapter can be described in terms of the equations above. Characterizing the steady state involves deriving equation (2). Explaining gradual adjustment involves deriving an error correction coefficient in equation (1) that is less than 1 and, ideally, close to 0.03.

With respect to issues that I do not address, perhaps the most important is the role of the inflation term π in equation (1). The theory is essentially one of *real* wage adjustment, whereas the regression describes *nominal* wages. The regression can be interpreted in real terms by moving π to the left hand side of equation (1). Interpreting this term as a proxy for expectations of current inflation, the regression then describes expected real wage adjustment. However, there are several issues relating to such an interpretation, which I do not explore. Other issues I do not explore include different rates of adjustment to different shocks, the role of the "other terms", the role of the residual and the precise functional forms of the empirical relationships.

4.3 Outline of the model and comparisons with previous research

My starting point is a matching model of the labor market, similar to that in Pissarides (1990). This model is often referred to as the "exogenous separations case" of Mortensen and Pissarides (1994). It assumes that unemployment is determined by flows in and out of employment. When an unemployed worker and a potential employer meet, both benefit from forming an ongoing relationship. In these conditions of bilateral monopoly, wages are determined through Nash bargaining.

This an attractive framework for considering wage determination as it allows for unemployment, bargaining, interactions between wages and other potentially important features. It implies a long run relationship determining wages that closely resembles equation (2). The minimum wage can be modeled as an outside option, consistent with the discussion of insecurity in Section 3.3. The model is consistent with, and explains, workers caring strongly about the wages paid on comparable jobs, which is an important element of models of overlapping contracts and also seems to be important in actual wage determination.

My work differs from others that have used this model in that I assume wages are governed by contracts. My approach is also unusual in that I explore implications for the wage data. With a few exceptions, such as Blanchard (1998), previous empirical work using matching models has focussed on the implications for labor market flows.

I take the existence of fixed duration contracts as given. It would be possible to motivate this assumption by the information costs of renegotiating wages (see Caballero, 1989; Ball and Mankiw, 1994 p141) or by signaling considerations that make state-contingent contracts unattractive. But there are other issues I wish to focus on.

By themselves, temporary contracts do not seem capable of explaining substantial wage inertia. The central tendency of estimates of contract duration seems to be about one year (Taylor, 1998). But my regressions and many theories imply that it takes *many* years for aggregate wages to adjust to shocks. That is, wage rigidity persists beyond the time required to renegotiate contracts. To explain this persistence, it seems necessary to examine further frictions. Accordingly, I focus on the staggering, or lack of synchronization, of contracts. The literature surveyed by Taylor (1998) suggests that the staggering of contracts may be more important than their existence.

Staggering is often appealed to as a justification for assuming inflexible aggregate wages. Examples include Brayton, Levin, Tryon and Williams (1997 p64), Ball and Mankiw (1994 p140) and Romer (1996 Ch6B). The argument, due to Taylor (1979, 1980), takes as given that individual wages are fixed for discrete periods of time. When it comes time for them to be renegotiated, they take other predetermined wages into account. New wage contracts can be modeled as a weighted average of a forward-looking equilibrium and the predetermined wages prevailing elsewhere. So inertia in one contract is passed on to other contracts. When later wages come up for renegotiation, they will be influenced by the backward looking elements of recent contracts. Thus the inertia persists beyond the time it takes all contracts to be renegotiated. Taylor (1980) describes this as a "contract multiplier": a limited degree of rigidity in individual wage contracts involves a much greater rigidity of aggregate wages.

This idea has been explored in a large number of papers. Taylor (1998) provides a survey. As many of these papers also attempt to explain the slow adjustment of prices and wages it seems appropriate to explain what my model contributes.

Most formal models of staggering focus on prices in product markets. This approach seems partly motivated by analytical considerations. For staggering to matter, price setters need to care about prices set elsewhere. This interaction is relatively simple to model if one assumes monopolistic competition, with the demand for a firm's product depending on relative prices. Examples include Chari, Kehoe and McGrattan (1996), Bergin and Feenstra (1998), Kiley (1999), Blanchard and Fischer (1989), and Romer (1998). In contrast, I explore staggering of wages rather than prices. This is interesting partly because of the central role played by wage rigidity in many theories of macroeconomic fluctuations and because wage rigidity may be more important than price rigidity.

Other authors have examined wage staggering but have done so assuming that the labor market is also monopolistically competitive. Examples include Ascari (1998), Erceg (1997) and Kim (1996). However, this seems implausible. One rarely observes workers, or even unions, unilaterally setting wages. Estimates of the bargaining power of workers, discussed below, are low – suggesting the labor market more closely resembles monopsony than monopoly. In contrast, I assume that wages are determined by bargaining. This seems more realistic. It has greater generality – encompassing monopolistic competition as a special case. And it explicitly allows for job search. By relating wage adjustment to unemployment (defined as people looking for work), bargaining models can shed light on the Phillips curve, the focus of many discussions of aggregate nominal rigidity.

A third strand of the literature on overlapping contracts assumes exogenous behavioral rules with no specific market structure. Taylor (1980) and the basic model of Taylor's (1998) survey are prominent examples. My approach differs in that I make the preferences and constraints facing individual price setters explicit, for the reasons noted at the opening of this chapter.

Another difference between my model and that of Taylor concerns the interaction between wages. In Taylor's model, this interaction is indirect, operating through the price level. In contrast, interaction between wages emerges directly from matching models: if a worker were not employed at a given firm, he could be earning another wage; the value of this alternative will be taken into account through bargaining. Although an effect through prices could be present, a direct concern with relative wages seems more intuitive and consistent with the original motivation of the literature on overlapping contracts (see for example, Taylor, 1980 p2).

4.4 A simple model

The basic matching model I use is described and explained in Pissarides (1990), Rogerson (1997) or Romer (1998). Given this, my discussion focuses on elements that are new.

There are two types of agent, workers and firms; both of whom have preferences that can be represented as

$$utility_t = \sum_{t=0}^{\infty} \delta^t income_t \tag{3}$$

where $income_t$ represents income at time t and δ represents a discount factor.

A worker can be unemployed with utility U and income b or employed at one of n jobs. Denote the utility and wage of a worker in the j'th job as W_j and w_j respectively, where j = 1 to n. Jobs are indexed by the time at which their wage is negotiated. That wage is then fixed for n periods. So w_1 is negotiated in periods 1, n+1, 2n+1 …. and w_j is negotiated in periods j, n+j, 2n+j, …

The timing of wage negotiations is not affected by when a worker starts at a job (or anything else). This assumption seems appropriate for describing collectively determined contracts. A steelworker who ratifies a three-year contract does so recognizing that, if he were to lose his job, he might find work as say an autoworker on a previously negotiated contract. Hence the auto contract is taken into account in negotiating the steel contract. These flow-ons, described as "pattern bargaining" or "wage contours" are often emphasized in institutional descriptions of wage bargaining.

An alternative assumption would be that the wage is initially negotiated at the time of a match. This seems to more naturally describe individually negotiated wages.

However, it implies that the alternative wage at which a worker might be reemployed is flexible rather than pre-determined. The relevant contracts do not overlap. This alternative assumption would not give rise to the interdependence and inertia in wages that I am interested in exploring. A more realistic and complicated model would allow for different types of contracts, including those negotiated at the time of a match. This would presumably exhibit less sluggish wages than my model.

The probability that an unemployed worker finds a job and becomes employed next period is p_t. I initially assume that this probability is a constant, then allow it to vary in later sections. The constant probability that an employed worker loses his job and becomes unemployed next period is *s*. Let an *i* subscript relate to an individual worker and match and a *t+1* subscript relate to values expected next period. Then workers' value functions can be written as:

$$W_{ijt} = w_{ijt} + \delta [sU_{t+1} + (1-s)W_{ijt+1}] \tag{4}$$

$$U_t = b_t + \delta [\Sigma_j(p_t/n)W_{jt+1} + (1-p_t)U_{t+1}] \tag{5}$$

For some purposes, it is convenient to have explicit expressions for the average worker. Summing equation (4) over *i* and *j* gives

$$\overline{W}_t = \overline{w}_t + \delta [sU_{t+1} + (1-s)\overline{W}_{t+1}] \tag{6}$$

where $\overline{W} = (1/n)\,\Sigma_j W_j$ is the expected payoff from a match (an argument in (5)) and $\overline{w} = (1/n)\,\Sigma_j w_j$ can be referred to as the "prevailing wage".

The utility of a match to a firm is *J* and its instantaneous payoff is output, *y*, less the wage. That is, I assume perfect competition and constant returns to labor, the sole factor of production. Matches end with probability *s*, after which the firm gets nothing.

$$J_{ijt} = y_t - w_{ijt} + \delta(1\text{-}s)J_{ijt+1} \tag{7}$$

A firm can always try to hire more workers, so becoming unmatched does not make searching any easier. In contrast, a worker can only match with one firm at a time – so becoming unmatched enables him to search. Because of this, the value of being unmatched is included in the worker's utility function (4), but not that of the firm (7). (As the value of being an unmatched firm is zero in equilibrium, this assumption is not important). This means that workers directly care about wages on other jobs while firms do not.

Wages for the i'th job of type j in periods $t = j, n+j, 2n+j, \ldots \infty$ are assumed to be determined by maximization of a generalized Nash product:

$$w_{ijt} = \text{argmax}\ (W_{ijt} - U_t)^{\beta} J_{ijt}^{1-\beta} \tag{8}$$

subject to the constraint

$$w_{ijt} = w_{ijt+1} = w_{ijt+\ldots} = w_{ijt+n-1} \tag{9}$$

The parameter β represents an index of workers' bargaining power (beyond that captured by the threat point U). Bargainers are assumed to take into account that the current bargain affects their wage for the next $n\text{-}1$ periods but not wages after that nor wages elsewhere in the economy. The probability of returning to the original job after a spell of unemployment is assumed to be zero. So the maximization is with respect to w_{ijt}, holding $\bar{w}_t$ constant. Substituting (9) into (4) and (7), then these equations into (8), differentiating, then canceling $\sum_{k=0}^{n-1}\delta^k(1-s)^k$ from both sides gives the first order condition:

$$(1\text{-}\beta)\ (W_{ijt} - U_t) = \beta J_{ijt} \tag{10}$$

In intervening periods, when $t \neq j,\ n+j,\ 2n+j,\ \ldots\ \infty$ the first order condition (10) need not hold and w_{ijt} is instead determined by a previously negotiated contract, as in (9).

The simplest path of wages that satisfies these conditions is the "steady state", when all variables are constant, so the j and t subscripts can be dropped. Substituting the value functions into (10) and solving gives

$$w_i = \beta y + (1-\beta)[\xi b + (1-\xi)\,\overline{w}\,] \tag{11}$$

$$\text{where } \xi = \frac{1-\delta(1-s)}{1-\delta(1-s-p)}$$

In steady-state, individual wage bargains will be an average of match productivity and the workers' outside opportunities, weighted by the bargaining parameter, β. The workers' outside opportunities (the term in square brackets) in turn are a weighted average of unemployment benefits and the prevailing wage, where the weight ξ depends on transition probabilities. When the probability of finding work, p, is high, the prevailing wage receives a high weight.

For parameters I use and discuss in the following section (converted to a monthly basis), $\beta = 0.1$, $\delta = 0.99$, $s = 0.06$ and $p = 0.44$, equation (11) implies

$$w_i = .1y + .12b + .78\overline{w} \tag{12}$$

The high weight on $\overline{w}$ means that individual wage setters care strongly about other wages. Blanchard (1998) makes this point with a similar model. This is a key element of overlapping contracts models. It also helps to explain why wages are staggered rather than synchronized (Ball and Cecchetti, 1988). And it coincides with casual observation of wage negotiations.

As all bargains solve the same problem, the i subscript can be dropped. Setting $w_i = \overline{w}$ gives an equation for the aggregate steady state wage:

$$w = \Psi y + (1-\Psi)\, b \qquad (13)$$

$$\text{where } \Psi = \frac{\beta[1-\delta(1-s-p)]}{\beta[1-\delta(1-s-p)]+(1-\beta)[1-\delta(1-s)]}$$

In Section 4.8, I consider a more complicated steady state which includes the minimum wage and the unemployment rate explicitly, as in equation (2). However the steady state of equation (13) provides a simpler context in which to first discuss dynamic adjustments.

4.5 Transition to the steady-state

To illustrate changes in wages over time, I first consider a special three-period case that yields an algebraic solution, then a T-period case that I solve numerically. I initially assume that the probability of finding a job is constant, then relax this assumption in Section 4.7, which allows for feedback effects from wages to job creation.

4.5.1 A three-period case

Consider firms and workers for whom $j = 1$. Suppose that they negotiate a wage, w_{1T-2}, at period $T-2$ that lasts for two periods, so $w_{1T-1} = w_{1T-2}$. Then, at period T, the economy is expected to reach the steady state, perhaps because all negotiations become synchronized. To further simplify, set $b = 0$, assume y is constant and assume no wage is being renegotiated at $T-1$. The firms' utility function can be written as:

$$J_{1T-2} = y - w_{1T-2} + \delta(1-s)J_{1T-1}$$

$$= y - w_{1T-2} + \delta(1-s)[\, y - w_{1T-2} + \delta(1-s)J_{1T}\,]$$

$$= (y - w_{1T-2})[1+\delta(1-s)] + \delta^2(1-s)^2 J_{1T} \qquad (14)$$

The corresponding expression for the worker's surplus is complicated by the weight placed on other jobs. Subtracting (5) from (4) gives

$$W_{1T-2} - U_{T-2} = w_{1T-2} + \delta[(1 - s)(W_{1T-1} - U_{T-1}) - \Sigma^n_{j=1}(p/n)(W_{jT-1} - U_{T-1})\,] \qquad (15)$$

$$= w_{1T-2} + \delta[(1 - s - p/n)(W_{1T-1} - U_{T-1}) - \Sigma^n_{j=2}(p/n)\,(W_{jT-1} - U_{T-1})\,]$$

$$= w_{1T-2} + \delta(1-s-p/n)w_{1T-2} - \delta(p/n)\,\Sigma^n_{j=2}w_{jT-1} + \delta^2(1-s-p)^2\,\Sigma^n_{j=1}(W_{jT} - U_T)/n$$

Let $w_0 = \dfrac{\sum_{j=2}^n w_{jT-1}}{n-1}$ represent the average predetermined wage and $W_T = \dfrac{\sum_{j=1}^n W_{jT}}{n}$ represent the workers steady state utility (which is equal for all j). So

$$W_{1T-2} - U_{T-2} = w_{1T-2}[1+\delta(1-s-p/n)] - \delta p(n-1)/n\, w_0 + \delta^2(1-s-p)^2\,(W_T - U_T) \qquad (16)$$

The first order condition when w_{T-2} is negotiated is

$$(1-\beta)\,(W_{1T-2} - U_{T-2}) = \beta J_{1T-2} \qquad (17)$$

Substitute (14) and (16) into this; then express J_T, W_T-U_T and y as multiples of the steady state wage w_T, and rearrange. This gives the negotiated wage as a weighted average of predetermined wages and a forward-looking equilibrium:

$$w_{T-2} = \chi w_0 + (1-\chi)w_T \qquad (18)$$

where

$$\chi = \frac{\frac{n-1}{n}(1-\beta)\delta p}{1+\delta(1-s) - \frac{(1-\beta)\delta p}{n}} \qquad (19)$$

or, for large n

$$\chi \approx \frac{(1-\beta)\delta p}{1+\delta(1-s)} \qquad (20)$$

Subtracting w_0 from each side of (18) gives a measure of the proportional rate of adjustment:

$$\frac{w_{T-2} - w_0}{w_T - w_0} = 1 - \chi \tag{21}$$

All the parameters determining χ lie between 0 and 1 except n, which exceeds 1. Given this, the rate of adjustment is positive, but less than 1. Wages adjust slowly towards the steady state when χ is high; which occurs if δ, s, p, and n are high and β is low. I discuss the sensitivity of the rate of adjustment to these parameters below.

Expressions similar to those above can be derived recursively for earlier periods, further in time from the steady state. These involve adding transformations of higher powers of $\delta(1\text{-}s)$ to the numerator and denominator of χ. However, the increasing complexity does not yield much insight. The sign of derivatives of the rate of adjustment with respect to different parameters remain as noted above. The rate of adjustment in earlier periods is lower, reflecting the longer period for which workers could be reemployed on a pre-determined wage. Whereas at $T\text{-}2$, a worker could be reemployed on a pre-determined wage for only one period, he could do so for up to $n\text{-}1$ periods when the horizon is not imminent. But exploring the stickiness of earlier bargains is more easily done numerically.

4.5.2 A T-period case

For a more general solution, when the steady state is distant, allow y, b and p to vary over time and expand (14) and (16) from period t to T.

$$J_{1t} = \sum_{k=0}^{T-1} \delta^k (1\text{-}s)^k (y_{t+k} - w_{1t+k}) + \delta^T (1\text{-}s)^T J_{1t+T} \tag{22}$$

$$W_{1t}\text{-}U_t = \sum_{k=0}^{T-1} [\mu_k(w_{1t+k} - b_{t+k}) + \varphi_k \Sigma^n_{j=2}(w_{jt+k} - b_{t+k})] + \mu_T(W_{1t+T} - U_{1t+T}) + \varphi_T \Sigma^n_{j=2}(W_{jt+T} - U_{t+T}) \tag{23}$$

where $\mu_0 = 1 \qquad \mu_{k+1} = \delta(1\text{-}s\text{-}p_{t+k}/n)\,\mu_k - (n\text{-}1)(\delta p_{t+k}/n)\,\varphi_k, \qquad k = 1,2,...,T\text{-}1$

$\varphi_0 = 0 \qquad \varphi_{k+1} = \delta(1\text{-}s\text{-}(n\text{-}1)p_{t+k}/n)\,\varphi_k - (\delta p_{t+k}/n)\,\mu_k,$

Substituting (22) and (23) into the first-order condition (10) and rearranging gives

$$\begin{aligned} w_{1t} = &\sum_{k=0}^{T-1} \beta\delta^k(1\text{-}s)^k y_{t+k} + (1\text{-}\beta)[\mu_k + (n\text{-}1)\varphi_k]\, b_{t+k} \\ &- \sum_{k=1}^{T-1} \{ [\beta\delta^k(1\text{-}s)^k + (1\text{-}\beta)\mu_k]\, w_{1t+k} + (1\text{-}\beta)\varphi_k \Sigma^n_{j=2} w_{jt+k} \} \qquad (24) \\ &- \beta\,\delta^T(1\text{-}s)^T J_{1t+T} - (1\text{-}\beta)\mu_T (W_{1t+T} - U_{1t+T})) - (1\text{-}\beta)\varphi_T\, \Sigma^n_{j=2}(W_{jt+T} - U_{t+T}) \end{aligned}$$

Wages negotiated at time t are a function of expected productivity and benefits (the first line), future wages, some of which are predetermined (the second line) and surpluses of workers and firms at a terminal date, $t+T$ (the third line). The weights on future variables reflect the probability that workers will be in the corresponding job, discounted for time. To determine w_{1t} then requires assumptions about wages expected in the future and terminal surpluses.

I assume that when wages come up for renegotiation, they are expected to solve the same problem as solved at time t. This is called model-consistent or rational expectations: all wage setters optimize in the expectation that other wage setters will behave similarly. This assumption seems implausible, given the prohibitive calculation costs involved. But other assumptions involve wage setters making systematic errors. As they learn, their behavior is presumably modified in the direction of the rational expectations solution. Rational expectations provide a stable solution to which other expectational assumptions converge and hence are a natural starting point for any analysis.

Although wages are interdependent, they are negotiated individually. The effect of any individual wage on others is negligible. Wage setters optimize taking other wages as given. So consistency between wages is a Nash equilibrium.

Formally, a (perfect-foresight, Nash) equilibrium can be defined as an infinite sequence of an n-vector of wages ($w_1, \ldots, w_j, \ldots w_n$) where at $t = j, n+j, 2n+j, \ldots \infty$, w_{jt} maximizes the Nash product (8) subject to (9), and at other times w_{jt} satisfies (9); utility is defined as in (4), (5) and (7); and $\boldsymbol{y}, \boldsymbol{b}, \boldsymbol{p}$, s, δ, β and an initial vector of wages, $\boldsymbol{w_0}$, are given.

If the horizon T is distant, the weights on terminal surpluses approach zero and these terms could be ignored. Then the equilibrium path of wages could be solved as a high order difference equation. A simpler and more flexible solution assumes a finite horizon; specifically that wages are expected to approximately equal their steady state levels after T periods. Then the equations above can be solved with matrix algebra.

The value functions given by (22) and (23) will also hold in later periods, though they will only determine wages when negotiations occur, and the horizon will shorten as T approaches. Setting these out in detail may clarify the structure of the solution. Although the notation may seem formidable, the economics is a straightforward application of the previous discussion. For brevity, assume that the horizon T is 6 periods ahead. This readily generalizes to more distant horizons. Then the equations given by (22) for $j = 1$ can be written as:

$$\begin{bmatrix} J_{1t} \\ J_{1t+1} \\ J_{1t+2} \\ J_{1t+3} \\ J_{1t+4} \\ J_{1t+5} \end{bmatrix} = \begin{bmatrix} 1 & \delta(1\text{-}s) & \delta^2(1\text{-}s)^2 & \delta^3(1\text{-}s)^3 & \delta^4(1\text{-}s)^4 & \delta^5(1\text{-}s)^5 & \delta^6(1\text{-}s)^6 \\ 0 & 1 & \delta(1\text{-}s) & \delta^2(1\text{-}s)^2 & \delta^3(1\text{-}s)^3 & \delta^4(1\text{-}s)^4 & \delta^5(1\text{-}s)^5 \\ 0 & 0 & 1 & \delta(1\text{-}s) & \delta^2(1\text{-}s)^2 & \delta^3(1\text{-}s)^3 & \delta^4(1\text{-}s)^4 \\ 0 & 0 & 0 & 1 & \delta(1\text{-}s) & \delta^2(1\text{-}s)^2 & \delta^3(1\text{-}s)^3 \\ 0 & 0 & 0 & 0 & 1 & \delta(1\text{-}s) & \delta^2(1\text{-}s)^2 \\ 0 & 0 & 0 & 0 & 0 & 1 & \delta(1\text{-}s) \end{bmatrix} \begin{bmatrix} y\text{-}w_{1t} \\ y\text{-}w_{1t+1} \\ y\text{-}w_{1t+2} \\ y\text{-}w_{1t+3} \\ y\text{-}w_{1t+4} \\ y\text{-}w_{1t+5} \\ J_{1t+T} \end{bmatrix} \tag{25}$$

With multi-period contracts, there is no bargaining in $(n\text{-}1)/n$ of these periods, so the corresponding rows can be disregarded. Suppose that $n=4$ and that negotiations for $j = 1$ occur at $t, t+4, \ldots$. Then $w_{1jt} = w_{1jt+1} = w_{1jt+2} = w_{1jt+3}$ and $w_{1jt+4} = w_{1jt+5}$. Imposing this and deleting those equations that do not affect wages then the above system becomes:

$$\begin{bmatrix} J_{1t} \\ J_{1t+4} \end{bmatrix} = \begin{bmatrix} \Sigma_0^3 \delta^k(1\text{-}s)^k & \Sigma_4^5 \delta^k(1\text{-}s)^k \\ 0 & 1+\delta(1\text{-}s) \end{bmatrix} \begin{bmatrix} y\text{-}w_{1t} \\ y\text{-}w_{1t+4} \end{bmatrix} + \begin{bmatrix} \delta^6(1\text{-}s)^6 \\ \delta^2(1\text{-}s)^2 \end{bmatrix} \begin{bmatrix} J_{1t+T} \end{bmatrix} \tag{26}$$

Similar systems describe jobs for $j =$ 2, 3, … n. Combining these gives a system describing the employers surplus from wages negotiated in each period $t, t+1, t+2, \ldots t+T$. The names of matrices are written in bold underneath:

$$
\underbrace{\begin{bmatrix} J_{1t} \\ J_{2t+1} \\ J_{3t+2} \\ J_{4t+3} \\ J_{1t+4} \\ J_{2t+5} \end{bmatrix}}_{\boldsymbol{J}} = \underbrace{\begin{bmatrix} \Sigma_0^3 \delta^k (1-s)^k & 0 & 0 & 0 & \Sigma_4^5 \delta^k (1-s)^k & 0 \\ 0 & \Sigma_0^3 \delta^k (1-s)^k & 0 & 0 & 0 & \Sigma_4^5 \delta^k (1-s)^k \\ 0 & 0 & \Sigma_0^3 \delta^k (1-s)^k & 0 & 0 & 0 \\ 0 & 0 & 0 & \Sigma_0^2 \delta^k (1-s)^k & 0 & 0 \\ 0 & 0 & 0 & 0 & 1+\delta(1-s) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{\boldsymbol{\alpha}} \underbrace{\begin{bmatrix} (y-w_1)_t \\ (y-w_2)_{t+1} \\ (y-w_3)_{t+2} \\ (y-w_4)_{t+3} \\ (y-w_1)_{t+4} \\ (y-w_2)_{t+5} \end{bmatrix}}_{\boldsymbol{y - w}}
$$

$$
+ \underbrace{\begin{bmatrix} \delta^6 (1-s)^6 \\ \delta^5 (1-s)^5 \\ \delta^4 (1-s)^4 \\ \delta^3 (1-s)^3 \\ \delta^2 (1-s)^2 \\ \delta (1-s) \end{bmatrix}}_{\boldsymbol{\alpha_T}} \underbrace{\begin{bmatrix} J_{t+T} \end{bmatrix}}_{\boldsymbol{J_T}} \qquad (27)
$$

The equations for the worker given by (23) have a similar structure, but with other jobs also being taken into account. The notation can be simplified by assuming that p_t is constant, though it varies in the numerical simulations below. The equivalent of equation (25) for the worker, (abbreviated to 3 periods, for space) is:

$$\begin{bmatrix} W_{1t}\text{-}U_t \\ W_{1t+1}\text{-}U_{t+1} \\ W_{1t+2}\text{-}U_{t+2} \end{bmatrix} = \begin{bmatrix} \mu_0 & \mu_1 & \mu_2 & \mu_3 \\ 0 & \mu_0 & \mu_1 & \mu_2 \\ 0 & 0 & \mu_0 & \mu_1 \end{bmatrix} \begin{bmatrix} w_{1t}\text{-}b_t \\ w_{1t+1}\text{-}b_{t+1} \\ w_{1t+2}\text{-}b_{t+2} \\ W_{1t+T}\text{-}U_{t+T} \end{bmatrix} + \Sigma^n{}_{j=2} \begin{bmatrix} \varphi_0 & \varphi_1 & \varphi_2 & \varphi_3 \\ 0 & \varphi_0 & \varphi_1 & \varphi_2 \\ 0 & 0 & \varphi_0 & \varphi_1 \end{bmatrix} \begin{bmatrix} w_{jt}\text{-}b_t \\ w_{jt+1}\text{-}b_{t+1} \\ w_{jt+2}\text{-}b_{t+2} \\ W_{jt+T}\text{-}U_{t+T} \end{bmatrix} \tag{28}$$

where μ and φ are shorthand for weights on own and other jobs respectively, as described by (23). Allowing for 4-period contracts, combining with systems for $j = 2, 3$ and 4, deleting the redundant equations and expanding the horizon T to 6 gives the equivalent of equation (27) for the worker:

$$
\underbrace{\begin{bmatrix} W_{1t}-U_t \\ (W_2-U)_{t+1} \\ (W_3-U)_{t+2} \\ (W_4-U)_{t+3} \\ (W_1-U)_{t+4} \\ (W_2-U)_{t+5} \end{bmatrix}}_{\boldsymbol{W-U}}
=
\underbrace{\begin{bmatrix}
\Sigma_0^3\mu_k & \Sigma_1^4\varphi_k & \Sigma_2^5\varphi_k & \varphi_3+\varphi_4+\varphi_5 & \mu_4+\mu_5 & \varphi_5 \\
\varphi_0+\varphi_1+\varphi_2 & \Sigma_0^3\mu_k & \Sigma_1^4\varphi_k & \varphi_2+\varphi_3+\varphi_4 & \varphi_3+\varphi_4 & \mu_4 \\
\varphi_0+\varphi_1 & \varphi_0+\varphi_1+\varphi_2 & \Sigma_0^3\mu_k & \varphi_1+\varphi_2+\varphi_3 & \varphi_2+\varphi_3 & \varphi_3 \\
\varphi_0 & \varphi_0+\varphi_1 & \varphi_0+\varphi_1+\varphi_2 & \mu_0+\mu_1+\mu_2 & \varphi_1+\varphi_2 & \varphi_2 \\
0 & \varphi_0 & \varphi_0+\varphi_1 & \varphi_0+\varphi_1 & \mu_0+\mu_1 & \varphi_1 \\
0 & 0 & \varphi_0 & \varphi_0 & \varphi_0 & \mu_0
\end{bmatrix}}_{\pi}
\underbrace{\begin{bmatrix} (w_1-b)_t \\ (w_2-b)_{t+1} \\ (w_3-b)_{t+2} \\ (w_4-b)_{t+3} \\ (w_1-b)_{t+4} \\ (w_2-b)_{t+5} \end{bmatrix}}_{\boldsymbol{(w-b)}}
$$

$$
+\underbrace{\begin{bmatrix}
\varphi_0 & \varphi_0+\varphi_1 & \varphi_0+\varphi_1+\varphi_2 \\
0 & \varphi_0 & \varphi_0+\varphi_1 \\
0 & 0 & \varphi_0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}}_{\pi_{\boldsymbol{0}}}
\underbrace{\begin{bmatrix} w_{2t-3}-b_{t-3} \\ w_{3t-2}-b_{t-2} \\ w_{4t-1}-b_{t-1} \end{bmatrix}}_{\boldsymbol{(w_0-b_0)}}
+\underbrace{\begin{bmatrix}
\mu_6 & \varphi_6 & \varphi_6 & \varphi_6 \\
\varphi_5 & \mu_5 & \varphi_5 & \varphi_5 \\
\varphi_4 & \varphi_4 & \mu_4 & \varphi_4 \\
\varphi_3 & \varphi_3 & \varphi_3 & \mu_3 \\
\mu_2 & \varphi_2 & \varphi_2 & \varphi_2 \\
\varphi_1 & \mu_1 & \varphi_1 & \varphi_1
\end{bmatrix}}_{\pi_{\boldsymbol{T}}}
\underbrace{\begin{bmatrix} W_{1t+T}-U_{t+T} \\ W_{2t+T}-U_{t+T} \\ W_{3t+T}-U_{t+T} \\ W_{4t+T}-U_{t+T} \end{bmatrix}}_{\boldsymbol{(W-U)_T}}
\qquad (29)
$$

The vector analogue of the first order condition is

$$(1-\beta)\,(\boldsymbol{W} - \boldsymbol{U}) = \beta \boldsymbol{J} \qquad (30)$$

Substituting (29) and (27) into this gives

$$(1-\beta)\,[\pi\,(\boldsymbol{w}-\boldsymbol{b}) + \pi_{\boldsymbol{0}}(\boldsymbol{w_0}-\boldsymbol{b_0}) + \pi_{\boldsymbol{T}}\,(\boldsymbol{W}-\boldsymbol{U})_{\boldsymbol{T}}\,] = \beta\,[\,\alpha\,(\boldsymbol{y}-\boldsymbol{w}\,)+\alpha_{\boldsymbol{T}}\boldsymbol{J_T}\,] \qquad (31)$$

Rearranging gives an expression for ***w***:

$$\boldsymbol{w} = [(1-\beta)\,\pi + \beta\alpha\,]^{-1}\,\{\,(1-\beta)\,[\pi\,\boldsymbol{b} - \pi_{\boldsymbol{0}}(\boldsymbol{w_0}-\boldsymbol{b_0}) - \pi_{\boldsymbol{T}}(\boldsymbol{W}-\boldsymbol{U})_{\boldsymbol{T}}\,] + \beta\,[\,\alpha\boldsymbol{y} + \alpha_{\boldsymbol{T}}\boldsymbol{J_T}\,]\,\} \quad (32)$$

The vector ***w*** represents an equilibrium path of wages, as they adjust from an arbitrary starting point, $\boldsymbol{w_0}$ towards a terminal steady state, given by (13). If we express the terminal surpluses in terms of the given parameters, then ***w*** is a linear function of exogenous variables. So, assuming the appropriate second-order conditions hold, this equilibrium path is unique.

4.6 Calibration

Analysis of equation (32) involves inverting a *T* x *T* matrix. This is too complicated to do algebraically. However, the properties of the adjustment path can be illustrated by supposing specific numerical values for the parameters.

I assume all contracts last one year, which is the central tendency of indirect estimates for the USA (Taylor, 1998). A large *n* is appropriate to more closely approximate continuous time and because the timing of wage negotiations appears to be uniformly distributed (Taylor, 1998). I set $n = 250$, approximately the number of working days in a year. Assuming an annual real interest rate of 4% gives a discount rate, δ, of $0.96^{1/n}$.

Taken literally, my estimated wage equation (2), in conjunction with equation (48) to be discussed below, implies a value of $\beta = 0.04$, implying that firms capture almost all the surplus from a match. However, my wage equation and the data on which it is based are not designed to reliably estimate this parameter. For example, other factors of production show little variation and I do not control for them. Microeconomic studies of the bargaining power of labor provide stronger evidence of a value of β near but slightly above zero. Low but positive estimates are obtained by Blanchflower, Oswald and Sanfey (1996), Blanchflower, Oswald and Garrett (1990), Nickell and Wadwhani (1990), Christofides and Oswald (1992) and Currie and McConnell (1992). Abowd and Lemieux (1993) argue that these estimates are biased downward, however their preferred estimate of 0.2 for unionized workers also implies little bargaining power overall, especially if one assumes that non-unionized workers will be relatively less powerful. I assume $\beta = 0.1$.

A relatively broad measure of transition rates seems appropriate to my model – encompassing movements outside the workforce and from one firm to another, but excluding temporary layoffs and within-firm movements. Using records from state unemployment insurance systems, Andersen and Meyer (1994) estimate that the quarterly separation rate on this basis is about 17 per cent a quarter – or 0.3 per cent on a daily basis [$0.003 = 1- (1-.17)^{4/250}$]. The short duration of jobs implied by this high separation rate contrasts with the long duration of contracts.

In steady state, the flow of hires will equal that of separations. However, to express this as a hazard rate requires estimating the stock from which hires are drawn, which is not simple. One possible measure is the number of workers who report they "want a job", sometimes called the "augmented unemployment rate". According to Blanchard and Diamond (1990 Figure 1), this averaged 11 per cent of the adjusted workforce from 1968 to 1986. Dividing the daily flow of hires (0.3 per cent) by this implies that 2.7% of those wanting work would find it every day. This implies a monthly hazard of 44% and a quarterly hazard of 82%.

Movements from one job to another are included in the numerator of p though they are not specifically counted in the denominator. The reason is that these movements involve an instantaneous transition through unemployment. They represent the limiting case, as p approaches 1. Nevertheless, these transitions play the same role as those that involve a non-zero spell of unemployment. Technically, my model assumes these transitions involve a day of unemployment, which seems an innocuous approximation, and better than assuming these transitions do not occur. This definition differs slightly from that used in the Current Population Survey, where a movement from one job to another is counted as a spell of unemployment only if the gap between jobs lasts throughout the survey's reference week.

Considerations of heterogeneity suggest that a value of 2.7% for p may be too low. Conceptually, p is the probability that those in employment could find alternative employment should their current job end. If workers differ in the speed with which they find work then this will exceed the ratio of hires to unemployment – because those workers who quickly find work are underrepresented in the unemployment pool.

The following table sets out parameter values including those which are normalized.

Table 4-1: Baseline parameter values

Parameter	Description	Value
δ	discount rate	$0.96^{1/n}$
s	separation rate	0.3%

p	probability of finding a job	2.7%
β	labor's bargaining power	10%
y	productivity	100
b	unemployment benefits	10
n	number of jobs	250
T	Time horizon	2000

The calibrated model can determine the response of wages for any change in the exogenous variables, including shocks that are temporary or anticipated. Figure 4.1 shows the transitional path of wages in response to one of the simplest experiments – an unanticipated permanent increase in productivity to 110, holding other parameters constant. The thin black line shows newly negotiated wages; the thick gray line shows the average wage. This experiment illustrates some general properties of the model. Adjustment to other shocks would be slightly different, but to sensibly address different adjustment paths to different shocks would require a more complicated model.

<u>**Figure 4.1: Adjustment path of wages**</u>

Following unanticipated increase in productivity

Adjustment to the increase in productivity comes in two stages. On the day of the shock, there is a large jump in newly negotiated wages, which move 50% of the distance towards the new steady state. The remaining adjustment is gradual, protracted and approximately linear, with about 0.8% of the remaining ‘disequilibrium’ being removed each period. That is, the path of newly negotiated wages can be described as

$$\Delta w_t = 0.5\Delta w_t^{ss} + .008(w_{t-1}^{ss} - w_{t-1}) \tag{33}$$

where w_{t-1} represents wages negotiated the previous day, Δw_t represents $(w_t - w_{t-1})/w_{t-1}$ and w_t^{ss} is the steady state level of wages given by (13) for the parameter values taken at period *t*. Wages approach the steady state asymptotically, consistent with assuming a finite horizon to approximate terminal conditions.

Average wages, of course, lag behind newly negotiated wages. The average wage moves 75% of the distance to the new steady state in the first year, then 86% of the remaining distance each following year (so 96% of the total adjustment is complete after two years). This rate of adjustment is gradual and approximately linear, as implied by the regressions in Section 4.2. However, the speed of adjustment is much faster. I discuss this difference below.

Because wages take longer to adjust than they do to renegotiate, there is a contract multiplier. However, it is small. The sluggishness of aggregate wages is mainly due to the existence of one year contracts; the staggering of these contracts plays a minor role. Because I take the existence of contracts as given, my model does not explain the major source of wage inertia.

Variations in the underlying parameters result in changes in the speed of adjustment, many of which are in line with the equation for the three-period model (19). The rate of adjustment is not very sensitive to variations in the discount rate, δ, or the separation rate, s. Nor is it sensitive to variations in n or T when these are large. However, it increases discernibly when either of these variables becomes small, as would be contemplated in algebraic exercises.

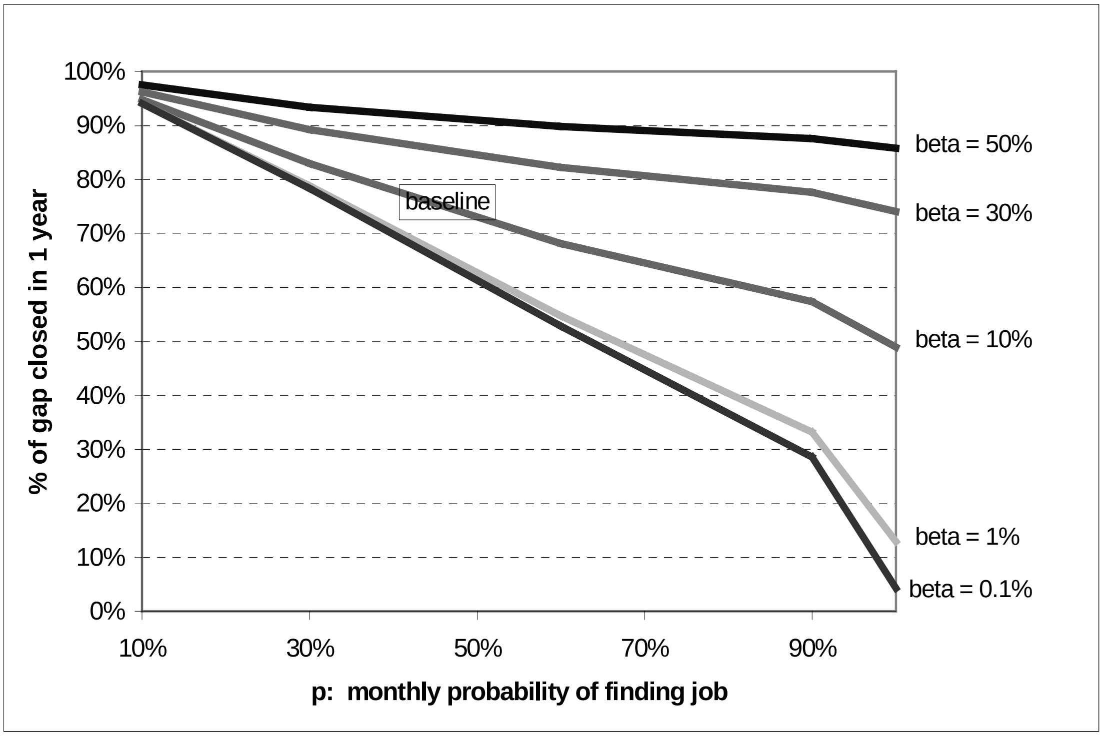
There are two reasons why inertia depends on the degree of staggering. At any point in time the fraction of wages that is predetermined and the fraction of the year for which some previously negotiated wages remain fixed both equal $(n-1)/n$. The three-period expression for the rate of adjustment (19) reflects the first of these arguments, though not the second. Both considerations give the past a greater weight as n increases.

The most interesting variations in the speed of adjustment concern those due to changes in the probability of finding another job, p, and the bargaining power of labor, β. The rate of adjustment decreases with p and increases with β. Wages are strongly influenced by previously negotiated contracts when the probability that the worker may end up on those contracts is high and when the wage is closely tied to the worker's outside opportunities.

Neither p nor β is reliably calibrated. Figure 4.2 below shows how much of the total adjustment in average wages is achieved after one year for differing values of p and β. The one year benchmark reflects the time it takes contracts to be renegotiated; so if nearly all the adjustment is complete after a year, the contract multiplier is small. For ease of comparison with other studies, I convert p to a monthly basis, $p^{monthly} = 1- (1-p)^{250/12}$.

Figure 4.2: Sensitivity of rate of adjustment

for variations in β and *p*



Over a wide range of parameters, including the baseline (marked in the figure), wages adjust fairly quickly, with most of the adjustment complete within a year. Any inertia in wages is attributable more to the existence of annual contracts than to their staggering. However, for parameter values that might be considered possible but unlikely, there is substantial wage inertia. For example, at $p^{monthly}$ = 90% and β = 1%, only 33% of the adjustment occurs in the first year, then 41% of the remaining adjustment in each following year. At extreme parameter values (the bottom right corner), wages are nearly rigid. I compare these predictions with empirical estimates in Section 4.9.

4.7 Endogenous vacancies

As wages change, so will firms' incentive to post vacancies. As vacancies adjust, so will the ease of finding work and hence the weights on future jobs. In this section, I take these feedback effects into account. Again, this follows Pissarides (1990).

I assume that an employer with a vacancy does not know when his match would be negotiated. It is equally likely to be at any of the j = 1 to n jobs, so the firm expects the average payoff from a match, $\bar{J} = 1/n(\Sigma_j J_j)$. This unnatural assumption simplifies the analysis, generating a uniform distribution of vacancies of different negotiation dates. Otherwise, the most profitable vacancies would drive others from the labor market.

Let the cost of a having a vacancy be c and the probability of finding a worker be q_t. Then the value of having a vacancy is

$$V_t \quad = - c + \delta[q_t \bar{J}_{t+1} + (1-q_t) V_{t+1}] \tag{34}$$

Vacancies are created, reducing q_t, until the return from doing so equals the cost, so that

$$V_t = 0 \tag{35}$$

(35) is expected to also hold at period $t+1$, implying:

$$q_t = c \ / (\delta \bar{J}_{t+1}) \tag{36}$$

To calculate the effect of wages on vacancy creation suppose initially that p_t is given so that a path of wages is determined, as in (32). Given wages, a path of firm surpluses, $\boldsymbol{J}$ can then be determined back from the terminal date, using (7). This is then averaged across jobs. Given expectations of $\boldsymbol{J}$, vacancies adjust until the probability of finding a worker, q_t, reflects the expected profit from search, as in (36).

q_t can then be related back to p_t through a matching function. Assume that x, the number of successful matches of firms with unemployed workers is a Cobb-Douglas function of the level of unemployment, u and the level of vacancies, v. Specifically:

$$x = x_0 u^{\eta} v^{1-\eta} \tag{37}$$

where x_0 and η are technological parameters. On average, the probability of vacancies finding a match equals the proportion of vacancies finding a match, or $q = x/v$. Substituting this into the matching function gives

$$q = \frac{x}{v} = \frac{x_0 u^{\eta} v^{1-\eta}}{v} = x_0 \left(\frac{v}{u} \right)^{-\eta} \tag{38}$$

The ratio v/u is a measure of the tightness of the labor market. Equation (38) simply says that as the labor market tightens, the chance of finding a worker, q declines. A similar argument shows the probability of finding a job, p, as increasing when the labor market tightens. Specifically

$$p = x_0 \left(\frac{v}{u} \right)^{1-\eta} \tag{39}$$

Combining (39) and (38) gives p_t as a function of q_t:

$$p_t = x_0^{1/\eta} q_t^{(\eta-1)/\eta} \tag{40}$$

This path of p_t can then be used to calculate a new set of weights on future wages. From which a new weighting matrix π can be calculated. This differs from the matrixes set out in (28) and (29) in that each row is no longer a copy of the row above. A new vector of wages $\boldsymbol{w}$ can then be calculated. Iterating on this routine enables wages and transition probabilities to be simultaneously determined. A new equilibrium could then be defined as sequences of wages, unemployment and vacancies, satisfying the above equations. (Though I have not made the determination of unemployment explicit above).

To illustrate, Table 4-2 shows some additional parameter values. These parameters determine p_t, which is now endogenous.

Table 4-2: More parameter values

Parameter	Description	Value
η	elasticity of matches w.r.t. unemployment	0.4
x_0	scales matching function	0.02
c	recruiting cost	783

A value of η = 0.4 is suggested by Blanchard and Diamond (1990). x_0 and c are then chosen so to give steady state transitions of p = 2.7% and q = 5%, the latter being consistent with a mean duration of vacancies of 3 weeks (Burdett and Cunningham, 1998).

Again, I perturb the system with an increase in productivity to 110. This increases profitability and hence vacancies and the probability of finding a job jumps from 2.7% to 3.1%. Then as wages rise, these effects are partially unwound and p_t declines to near its new steady state of 2.9% in about a year.

The variations in p_t have two offsetting effects on the speed of adjustment. The tightening of the labor market raises the target to which wages converge, putting upward pressure on wages. Because the initial tightening exceeds that in the long run, the adjustment of wages is brought forward. Partially offsetting this, the increased probability of finding a job increases inertia, as discussed in Section 4.6.

Under the parameters assumed above, the first of these effects is greater, though the overall effect is small. The average wage has moved 80 per cent of the distance to the new equilibrium after one year, compared to 75 per cent in the baseline. That is, allowing for feedback effects from wages to vacancies makes it slightly more difficult to explain the inertia in wages.

4.8 The role of minimum wages

Previous sections discussed the gradual adjustment of wages towards their steady state, where the steady state was a weighted average of unemployment benefits and productivity. This section examines some more of the determinants of the steady state.

The most controversial and unusual feature of equations (1) and (2) is that the level of the minimum wage has an important effect on aggregate wages. This effect is clearer in the data than the effect of unemployment benefits or productivity. In this section I provide some minor modifications to the steady state model of Section 4.4 to show how and why this might be the case.

If the minimum wage was thought to increase the natural rate of unemployment by reducing the demand for unskilled labor, it would be appropriate to model the productivity of labor as heterogeneous. Flinn (1998) develops a model along these lines. However, this effect is disputed by many economists and policy makers. So, instead, I give the minimum wage a "safety net" role. It influences wages by forming part of the worker's outside option. This approach is both simple and consistent with evidence discussed in Section 3.3 that the low wage outcomes over the 1990s were due to worker insecurity.

The model below differs from that in the previous section in that the alternative wage at which a worker might be reemployed is now determined by statute rather than by contract. This involves two further changes. Because this alternative wage is low, I assume that workers employed at the minimum wage continue searching for "good" jobs. Because the alternative wage is a policy instrument, I now assume it is exogenous.

Modify the model of Section 4.4 to allow for a third state that workers might be in, employment at a "bad job" with utility *M*. Suppose that these jobs pay the minimum wage *m*, which exceeds the unemployment benefit, *b*. Then *w* can be redefined as the negotiated wage from a "good job", which provides utility *W*.

Let p_m and p_w represent the probability of finding a bad job and good job respectively. On-the-job search is assumed to be as effective as unemployed search. This implies that workers accept all higher paying job offers and that there are no direct transitions from good jobs to bad jobs. Assume a steady state, so *t* subscripts can be dropped. Then value functions can be written as:

$$U = b + \delta [p_m M + p_w W + (1-p_m - p_w) U] \tag{41}$$

$$W = w + \delta [sU + (1-s) W] \tag{42}$$

$$M = m + \delta [p_w W + sU + (1-p_w - s) M] \tag{43}$$

The utility function of firms paying market wages, the wage bargaining equation, and consequently the first-order condition are the same as in Section 4.4, reproduced here for convenience, without subscripts:

$$J \quad = \; y \; - w \quad + \delta(1\text{-}s)J \tag{7)*}$$

$$w = \text{argmax}\,(\,W - U\,)^{\beta}\,J^{1-\beta} \tag{8)*}$$

$$(1\text{-}\beta)\,(\,W - U\,) = \beta J \tag{10)*}$$

Then comes some algebra. Multiplying both sides of (10)* by $1 - \delta\,(1 - s)$, then using (42) and (7)* gives:

$$w = (1\text{-}\beta)\,(1-\,\delta)U + \beta y \tag{44}$$

Substituting for $(1-\,\delta)U$ using (41), rearranging, substituting for $W\text{-}U$ using (10)*, subtracting (41) from (43) and rearranging further gives:

$$w = \;(1\text{-}\beta)\,[(1\text{-}\phi)b + \phi m] + \beta\,[y + p_w\delta\,J] \tag{45}$$

where $\phi = p_m\,/\,[1\text{-}\;\delta(1\text{-}\lambda\text{-}p_w\,\text{-}p_m)\,]$

Let q_w represent the probability that a firm paying market wages might find an unemployed worker, p_w represent the probability of an unemployed worker getting a good job and x_w represent the number of successful matches of good jobs with unemployed workers. Then $p_w = x_w / u$ and $q_w = x_w / v_w$. The ratio of these is

$$p_w/q_w = \; v_w/\,u. \tag{46}$$

In contrast to Section 4.7, the form of the matching function determining x_w does not need to be specified.

Assume that the labor market for good and bad jobs is segmented, so that matching in one market does not affect matching in the other. As in Section 4.6, assume that vacancies for good jobs are created, reducing q_w, until the return from doing so equals the cost. Then a corresponding version of the free-entry condition can be written:

$$\delta J = c / q_w \tag{47}$$

Then substituting (47) and (46) into (45) gives

$$w = (1-\beta)\,[(1-\phi)b + \phi m] + \beta\,[y + c\, v_w / u\,] \tag{48}$$

where ϕ is given in (45). The negotiated wage is an average of "opportunity cost" elements and gain sharing elements, weighted by bargaining power. The opportunity cost elements, b and m, can be interpreted as a safety net. If workers lose a good job then they get b or m until they succeed in finding another good job. If either of these variables decline relative to wages, then the costs of job loss increase and wages adjust downwards. The relative importance of unemployment benefits and the minimum wage within the safety net reflects the relative time spent in these respective activities. The return to a successful match is its productivity plus the saving in future recruitment costs, which increases as the labor market tightens. Wages reflect the difficulty with which workers can be replaced.

Equation (48) closely resembles the implied steady state in my regression equation (2). Specifically, wages are an increasing function of unemployment benefits, the minimum wage, productivity and the tightness of the labor market. The upwards pressure that say minimum wages place on market wages can be offset by higher unemployment. In quantitative terms, the equation implies that the coefficients on b, m and y should sum to 1. In my regression, the unconstrained sum of these coefficients is very close to this, specifically 1.02, with a standard error of 0.05. Equation (48) implies that the coefficient on y, 0.04, can be interpreted as β, the bargaining power of labor. This estimate is consistent with microeconomic studies, mentioned in Section

4.6, that have directly estimated this parameter.

4.9 Areas for further study

The description of the steady state in equation (48) and the theory of gradual wage adjustment outlined in the sections 4.6 and 4.7, provide an interpretation of many of the features of the wage data noted in Section 4.2. However, much remains to be explained.

One obvious difference between the steady state equation for wages (48) and that estimated in (2) is the specification of labor market tightness. In the theory, this is specified as the ratio of vacancies to unemployment. However, the (demographically adjusted) rate of unemployment fits the data better. Why this occurs is unclear. One possible reason is that vacancies are poorly measured, being proxied by help-wanted advertising. A related reason may be that the ratio of true vacancies to unemployment is strongly correlated with the unemployment rate. This would be consistent with shifts in the true Beveridge curve being small relative to movements along the curve. A related reason may be that demographic adjustments capture most shifts of the Beveridge curve.

A more fundamental difficulty with interpreting estimated wage equations as representing convergence to a long run relationship given by equation (48) is that the tightness of the labor market is endogenous. High wages have a negative effect on firms' incentive to post vacancies. Specifically, substituting (38) and (7)* into (47) implies that:

$$\left(\frac{v}{u}\right)^{\eta} = \frac{x_0\delta}{c[1-\delta(1-s)]}(y-w) \tag{49}$$

In contrast to (48), where the wage and the tightness of the labor market are positively related, here the relationship is negative. In equilibrium, (48) and (49) simultaneously determine these variables. However, in the data, (48) dominates. It is not obvious why this occurs. Presumably, shifts in (49) exceed those in (48). This could be because of variations in x_0, but is more plausibly because of some unspecified influence, such as aggregate demand, that is yet to be modeled. To minimize simultaneity problems in estimation the unemployment rate is lagged in equation (2). A better approach might involve instrumental variables, but that first requires an empirical understanding of vacancy determination.

Perhaps the most important difference between the theory and the econometrics concerns the rate of adjustment. With my baseline parameters and endogenous vacancies, 80 per cent of the total response of aggregate wages to a shock comes within a year, implying an average quarterly rate of adjustment of about 33 per cent. In contrast, the estimated wage equation (1) has a coefficient on the error-correction term of .03. This suggests that wages adjust to variables that enter only through the error-correction mechanism (that is, unemployment benefits) at a rate of 3 per cent a quarter or 12 per cent a year. There will be faster initial adjustment for those variables that also enter in change terms (such as the "other terms" in (1)). For example, about 70 per cent of the adjustment of nominal wages to a change in prices is complete in the first year. About 30 per cent of the total response to productivity and about 20 per cent of the response to a change in the minimum wage or unemployment are complete within a year. Thus the baseline parameters are roughly consistent with the relatively quick adjustment to prices but not with the very slow estimated response to say the minimum wage or unemployment.

This may mean that different parameter values would be appropriate. As discussed in Section 4.6, it is possible to generate very sluggish wages if workers perceive they can quickly regain employment elsewhere and if employers capture most of the surplus from a match. It is not clear, however, that these assumptions would be consistent with microeconomic evidence on these parameters.

Alternatively, one could modify the empirical estimates. The rate of adjustment towards the steady state is sensitive to the equation specification and imprecisely estimated, with a standard error slightly larger than the coefficient. The hypothesis that say, 25 per cent of the adjustment occurs within a year could not be clearly rejected. Furthermore, changes in specification, such as inclusion of lagged changes in wages would also imply faster adjustment. For example, in the FRB/US model (Brayton et al, 1996), wages are estimated to adjust towards their steady state level at a rate of about 35 per cent a year.

The third possibility is that important sources of inertia remain to be modeled. Possible examples include interactions with prices, recognition lags, other means of forming expectations and considerations of fairness. Because extensions along these lines would be interesting for other reasons, this seems to me to be the most promising approach.

In particular, learning effects may also help to explain the different speed of adjustment of wages to different shocks. Relevant information on consumer prices is publicly available with negligible cost and little delay. In contrast, information on a worker's outside opportunities is idiosyncratic and takes time and effort to process. This may help explain why wages adjust faster to prices than to say unemployment or other wages. It also provides a rationale for the time-dependent nature of contracts. Closely related, the difficulty of verifying firm-level productivity and price information may explain the apparent importance of proxies such as consumer prices and trend aggregate productivity.

My model assumes that prices are given. As such it describes economies with exogenously determined price levels, such as a region within a larger economy, or a country under the Gold Standard. The United Kingdom could be placed within the latter category between 1861 and 1913, the period to which Phillips (1958) initially fitted his famous curve. In that sense, my model can be seen as a "theory of the Phillips curve". In particular, it explains how the maximizing decisions of individual wage negotiators will give rise to a negative relationship between unemployment and the rate of change of wages.

To explain data for the USA, feedback effects from wages to prices need to be taken into account. These constitute the main channel of influence in Taylor's model of staggered wages. So an analysis of them would be interesting for academic reasons and presumably would help to account for the inertia in wages. And they are necessary for unemployment to affect inflation (and so to generate a Phillips curve defined that way). Explaining this effect would permit a microeconomic analysis of the NAIRU and help explore its relation to other "equilibrium" notions of unemployment. But all these extensions remain to be investigated.

There are of course many further differences between the patterns apparent in the data, as described by the estimated equations (1) and (2), and the theory of wage determination outlined above. Whether these should involve modifications to the econometrics, the parameters or to the theoretical model is an issue to be resolved by further research.

4.10 Conclusion

The behavior of aggregate nominal wages in the United States can be well described by a regression in which wages slowly converge towards their steady state level. This steady state is an increasing function of productivity, unemployment benefits and the minimum wage and a negative function of unemployment. The nature of the steady state can be explained by a bargaining model, in which the minimum wage and unemployment benefits provide “outside options” and unemployment makes workers difficult to find. Gradual adjustment towards the steady state can be explained by overlapping wage contracts.

When a model with these elements is calibrated to microeconomic information on the relevant parameters, it mimics some of the important features of the aggregate data. However, adjustment of wages is much faster in the calibrated model than in my regressions. In the previous section I suggested various ways in which this inconsistency might be removed. Perhaps the most interesting of these would be allowing for feedback effects from prices to wages.